2. V. L. Zhokhov and A. A. Krivenko, Dvigatelestroenie, No. 6, 43-45 (1989).
3. Z. R. Gorbis, Heat Transfer and Hydromechanics for Particle-Bearing Flows [in Russian], Moscow (1970).
4. M. E. Deich, V. I. Kiryukhin, G. A. Filippov, and O. A. Povarov, Teplotekhnika, No. 8, 20-23 (1974).
5. Bigger, Tabakoff, and Hamed, Proc. ASME, Power Machines Plant, 104, No. 1, 43-51 (1982).
6. M. F. Husslin and W. Tabakoff, J. Aircraft, 10, No. 7, 434-440 (1973).
7. S. L. Sow, Multiplase-System Hydrodynamics [Russian translation], Moscow (1971).
8. W. Goldschmidt, Collisions: Theory and Physical Properties of Colliding Bodies [Russian translation], Moscow (1965).
9. Yu. N. Krivenko, Hydromechanics [in Russian], Issue 16, Kiev (1970), pp. 57-62.
10. M. I. Zhukovskii, Aerodynamic Calculations on Axial Turbines [in Russian], Leningrad (1967).

EXTRUSION RHEODYNAMICS FOR A VISCOUS COMPRESSIBLE MATERIAL

L. S. Stel'makh, A. M. Stolin, and B. M. Khusid

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An analytic solution is derived for the consolidation and flow of a viscous compressible material used in plunger extrusion, which enables one to determine the density, velocity, and stress patterns in the specimen and in the rod.

Components are increasingly prepared from refractory powders by plunger extrusion, in which the material is extruded from a press mold through a die.

Good models exist for such extrusion for incompressible plastic and viscous materials [1-5] such as most polymer and metal systems. However, those models are sometimes inapplicable for refractory compressible powder composites. Such a material behaves as a viscous or viscoplastic body only over 1000°C, so external heating is used (hot extrusion [7, 8]). The latter method can give components from powders that do not press well. The rheological behavior and the combination of deformation and consolidation mean that such systems must be considered separately. The features are reflected in consolidation theory for a viscous compressible material, which can be based on a rheological approach [9, 10]. Numerous papers deal with axial compression for a viscous porous material [11-14]. Here we consider the extrusion of a viscous compressible material from a chamber via a slot. The model and the method are used with Lagrangian coordinates to obtain an analytic solution for the density and velocity distributions in the chamber and in the rod.

Model and Main Assumptions. We consider the flow of a viscous porous material from a cylindrical chamber bounded above by a moving piston. The initial length of the material is H_0, and the radius of the cross section is r_0. At the bottom of the chamber there is a circular hole with radius r_1 through which the material is extruded into a cylindrical guide of the same radius. The symmetry axis is taken as the z axis, whose positive direction is opposite to that of the piston motion. The origin $z = 0$ lies at the center of the exit cross section from the chamber. We neglect the friction on the walls of the chamber and guide cylinder and affects from bulk forces.

The flow region is divided into two parts: within the chamber between the piston at $z = H(t)$ and the exit section $z = 0_+$ and that within the guide between $z = 0_-$ and the free surface $z = L(t)$. The subscripts + and − indicate correspondingly that the $z = 0$ section relates either to the chamber or to the guide. We neglect perturbations in the two parts of the flow on passage from the chamber to the hole. The motion in each region is taken as steady-state and one-dimensional with one nonzero velocity component $v_z = v \neq 0$. If the

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viscosity μ_1 of the incompressible matrix in the powder composite is sufficiently large ($\mu_1 > 10^3$ Pa·sec), the flow rearrangement time near the exit hole with radius r_1 ($\sim\rho_1 r_1^2 \mu_1^{-1}$) in a real extrusion is small by comparison with the characteristic extrusion time ($\sim H_0|V|^{-1}$), and that assumption is justified. Usually, the extrusion is performed in a conical housing, which is not incorporated here. That assumption applies if the conical part is relatively short, $l \ll H_0$.

In a one-dimensional approach, the motion in the conical housing is characterized by two parameters: the relative density change and the hydraulic resistance, which is dependent on the applied force:

$$\frac{\rho(0_-,\ t)}{\rho(0_+,\ t)} = B(|\sigma_{zz}|_{z=0}|),$$

$$-S_1\rho_1\rho(0_-,\ t)\,v(0_-,\ t) = f(|\sigma_{zz}|_{z=0}|).$$

The shape of the housing and the rheological parameters govern B and f. We subsequently assume that B = 1, i.e., there is no ongoing consolidation in the housing, which is an acceptable approximation if the main consolidation occurs within the chamber. The $f(|\sigma_{zz}|)$ dependence for simplicity is taken as a power law: $f = k|\sigma_{zz}|^n$, where the parameters k and n may be derived from experiment.

The flow is described by the equations of motion and continuity together with the rheological formula, which can be put as [11-14]

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial z}(\rho v) = 0,\quad \frac{\partial\sigma_{zz}}{\partial z} = 0,$$
$$\sigma_{zz} = \left(\frac{4}{3}\mu + \xi\right)\frac{\partial v}{\partial z},\quad \sigma_{rr} = \sigma_{\theta\theta} = \left(-\frac{2}{3}\mu + \xi\right)\frac{\partial v}{\partial z}. \tag{1}$$

The boundary and initial conditions are

$$\sigma_{zz}|_{z=H(t)} = -P,\quad \sigma_{zz}|_{z=0} = 0,$$
$$\rho|_{t=0} = \rho_0(z).$$

The unknowns here are the relative density ρ and the velocity v, which are functions only of z and time t, while the shear viscosity μ and the bulk viscosity ξ have the following density dependence:

$$\mu(\rho) = \mu_1\rho^m,\quad \xi(\rho) = \frac{4}{3}\mu(\rho)\frac{\rho}{1-\rho}.$$

Mass-Balance Equations. Let M_1 and M_2 be the masses of material in the chamber and guide correspondingly, and then the total mass M_0 between the piston z = H(t) and the free surface z = −L(t) should be constant:

$$M_0 = M_1 + M_2 = S_0\rho_1\int_{0_+}^{H(t)}\rho(z,\ t)\,dz + S_1\rho_1\int_{-L(t)}^{0_-}\rho dz = \text{const}. \tag{2}$$

We use the initial conditions and the fact that $H(0) = H_0$, $L(0) = 0$ to get $M_0 = S_0\rho_1\int_0^{H_0}\rho_0(z)dz$. We check whether (2) can be met. We calculate $\partial M_0/\partial t$ as the derivative with variable upper limit and use (1) to show that

$$\frac{\partial M_0}{\partial t} = \frac{\partial M_1}{\partial t} + \frac{\partial M_2}{\partial t} = S_0\rho_1\rho(0_+,\ t)\,v(0_+,\ t) - S_1\rho_1\rho(0_-,\ t)\,v(0_-,\ t).$$

There is no density discontinuity on passage through z = 0, so $\rho(0_+,\ t) = \rho(0_-,\ t) = \rho(0,\ t)$. By virtue of the continuity, the amount of material leaving the chamber and the amount entering the guide are the same:

$$S_0\rho_1\rho(0,\ t)\,v(0_+,\ t) = S_1\rho_1\rho(0,\ t)\,v(0_-,\ t),$$

so $\partial M_0/\partial t = 0$ and (2) is met. The density assumption means that the change in cross-sectional area affects only the velocity, as for an incompressible liquid, and

$$v(0_+,\ t) = \frac{S_1}{S_0} v(0_-,\ t). \tag{3}$$

From (1), the longitudinal force does not vary along the chamber, so $\sigma_{zz} = -P$, and the hole resistance law is

$$\frac{dM_2}{dt} = -S_1\rho_1\rho(0,\ t)\, v(0_-,\ t) = kS_1P^n. \tag{4}$$

A similar approach is used in extrusion head theory [1]. From (3) and (4) we have

$$v(0_-,\ t) = -\frac{kP^n}{\rho_1\rho(0,\ t)},\quad v(0_+,\ t) = -\frac{S_1kP^n}{S_0\rho_1\rho(0,\ t)}. \tag{5}$$

The density $\rho(0, t)$ appears in (5) for the extrusion rate, and this has to be determined. We consider the consolidation in the chamber.

Lagrangian Coordinate System. We use Lagrangian coordinates: time t_L, which is taken as equal to the time t (with the subscript L subsequently omitted) and the mass coordinate $q = M/S_0\rho_1$, which is the relative mass of material between the variable section z and the free surface $z = -L(t)$:

$$q = \frac{M}{S_0\rho_1} = \int_0^z \rho(z,\ t)\,dz + \frac{S_1}{S_0}\int_{-L(t)}^0 \rho(z,\ t)\,dz.$$

We put $M_2(t) = S_1\rho_1 \int_{-L(t)}^0 \rho dz$ as the mass of material in the guide at time t. Then the relative total mass is governed by the coordinate q_0:

$$q_0 = \int_0^{H(t)} \rho(z,\ t)\,dz + \frac{M_2(t)}{S_0\rho_1}.$$

The (4) resistance law gives

$$q = \int_0^z \rho(z,\ t)\,dz + \overline{P}t,\quad q_0 = \int_0^{H(t)} \rho(z,\ t)\,dz + \overline{P}t. \tag{6}$$

Figure 1 shows the piston coordinates, the free surface, and the hole at the initial and current instants in terms of the z and q axes. The q ranges for the individual volumes in the chamber $q \in [\overline{P}t,\ q_0]$, have constant upper limit q_0 and time-varying lower limit $\overline{P}t$. That limit moves to the right with velocity $\overline{P}$. We consider any individual volume with coordinate q, which passes through the hole at time

$$t = q/\overline{P}. \tag{7}$$

The total extrusion time is: $t_e = q_0/\overline{P}$. The mass-coordinate range in the guide is $q \in [0,\ \overline{P}t]$. The length of this increases with time, so if we are interested in the characteristics of volumes passing through the hole at z = 0, one should use the q–t relation from (7), in which these quantities are not independent variables.

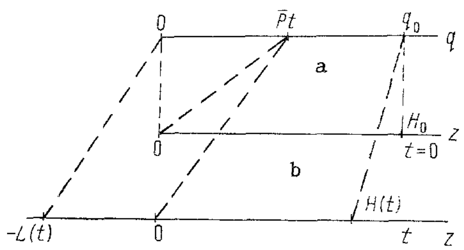

Fig. 1. Position of the $\overline{P}t$ boundary on the q-axis as a function of time and corresponding positions of the $-L(t)$ and $H(t)$ boundaries on the z-axis: a) at t = 0; b) at the current instant (t).

Formulation of Lagrangian Coordinates. In these, (1) becomes

$$\frac{\partial \rho}{\partial t} + \rho^2 \frac{\partial v}{\partial q} = 0, \tag{8}$$

$$\frac{\partial \sigma_{zz}}{\partial q} = 0, \tag{9}$$

$$\sigma_{zz} = \left(\frac{4}{3}\mu + \xi\right) \frac{\rho^{m+1}}{1-\rho} \frac{\partial v}{\partial q}. \tag{10}$$

To solve this, one needs to know the initial density distribution in q:

$$\rho(q, 0) = \rho_0(q) \tag{11}$$

and the boundary conditions. At z = H(t), with the force on the piston defined,

$$\sigma_{zz}|_{q=q_0} = -P, \tag{12}$$

and at the lower boundary

$$v|_{q=\overline{P}t} = v(0_+, t). \tag{13}$$

A difference from [12] is that here the velocity at the bottom of the chamber is not zero and is governed by the rate of extrusion into the guide.

We substitute for $\partial v/\partial q$ from (8) into (10) and use (9) and (12) to get the consolidation rate:

$$\frac{\partial \rho}{\partial t} = \frac{3}{4} \frac{P}{\mu_1} \frac{1-\rho}{\rho^{m-1}}. \tag{14}$$

A similar equation has been derived [12] for consolidation in a closed chamber, but there was a difference in that the Lagrange coordinate q was taken in the form of (6) and the range was different, which varied with time. The characteristic consolidation time is $t_* = 4\mu_1/3P$, and (14) can be rewritten as

$$\int_{\rho_0}^{\rho} \frac{\rho^{m-1}}{1-\rho} d\rho = \frac{t}{t_*}. \tag{15}$$

There is no difficulty in calculating

$$J(\rho) = \int_{\rho_0}^{\rho} \frac{\rho^{m-1}}{1-\rho} d\rho .$$

One can expand the integrand as a Taylor series, which converges rapidly for low values of the porosity ($\Pi = 1 - \rho \ll 1$), and calculate $J(\rho)$ approximately or derive an analytic expression for integer m. One thus gets $\rho(q, t)$ in the chamber for $\overline{P}t \le q \le q_0$.

We now derive the velocity distribution in the chamber. Equation (10) with (12) gives

$$\frac{4}{3} \mu_1 \frac{\rho^{m+1}}{1-\rho} \frac{\partial v}{\partial q} = -P.$$

We integrate this with (13) at the lower end of the chamber ($q = \overline{P}t$), to get

$$v(q, t) = v(\overline{P}t, t) - \frac{1}{t_*} \int_{\overline{P}t}^{q} \frac{1-\rho}{\rho^{m+1}} dq. \tag{16}$$

Here $v(\overline{P}t, t)$ means the velocity of the material for the Cartesian coordinate $z = 0_+$ before extrusion, for which (5) applies, which contains the density in the hole $\rho|_{z=0} = \rho(\overline{P}t, t)$, which is derived from (15) with $q = \overline{P}t$.

Density and Velocity Distributions in Guide. There is no ongoing consolidation in the guide on these assumptions, so in the region $0 \le q \le \overline{P}t$, corresponding to the guide, the density is not a function of time and is dependent only on q. Each volume with coordinate q passes through z = 0 at time $t = q/\bar{P}$, and attains its limiting density, with the further displacement along the guide not involving any more consolidation. The density distribution in the guide is derived from (15) with $t = q/\bar{P}$, in which $0 \le q \le \bar{P}t$. When one determines the speed of the extruded material, the above assumptions imply that there is no resistance to the motion in the guide: the friction in the guide is negligible and the stress at the free surface is zero, i.e., at $z = -L(t): \sigma_{zz} = 0$, which corresponds to the situation most often found with self-propagating synthesis extrusion for refractory materials, where the consolidation is largely completed in the chamber and the extrusion is that of a rigid material into air or into the guide, whose diameter is greater than that of the extruded rod. The rod moves with a variable velocity $v_L(t)$, which is determined by the velocity of the individual volume present in the hole at time t. The velocity in the guide is independent of q and is governed by the extrusion rate $v(\bar{P}t, t)S_0/S_1$, whose determination procedure has already been described.

<u>Particular Case m = 1.</u> A qualitative analysis may be based on taking this case separately, which corresponds to a linear relation between the shear viscosity and the density. Equation (15) gives the density distribution in the chamber:

$$\rho(q, t) = 1 - (1 - \rho_0(q))\exp(-t/t_*),\ \bar{P}t \leqslant q \leqslant q_0. \tag{17}$$

The density in the hole is given by (17) with $q = \bar{P}t$:

$$\rho(\bar{P}t, t) = 1 - (1 - \rho_0(\bar{P}t))\exp(-t/t_*). \tag{18}$$

We substitute for $\rho(q, t)$ from (17) into (16) to get the velocity distribution. If the porosity is low, the velocity is approximately

$$v(q, t) = v(\bar{P}t,\ t) - \frac{\exp\left(-\frac{t}{t_*}\right)}{t_*} \int_{\bar{P}t}^{q} \Pi_0(q)\, dq. \tag{19}$$

From (18), we have the density in the hole, and from (5) we get the velocity there:

$$v(\bar{P}t,\ t) = -\bar{P}\left[1 - (1 - \rho_0(\bar{P}t))\exp\left(-\frac{t}{t_*}\right)\right]^{-1}, \tag{20}$$

while the speed V of the piston is defined by (19) with $q = q_0$ and from (20) is put as

$$V = -\bar{P}\left[1 - (1 - \rho_0(\bar{P}t))\exp\left(-\frac{t}{t_*}\right)\right]^{-1} - \frac{\exp\left(-\frac{t}{t_*}\right)}{t_*} \int_{\bar{P}t}^{q_0} \Pi_0(q)\, dq. \tag{21}$$

We then determine the characteristics in the guide. The density in the extruded part of the material is found from (17) by putting $t = q/\bar{P}$:

$$\rho(q) = 1 - (1 - \rho_0(q))\exp\left(-\frac{q}{\bar{P}t_*}\right),\ q \in [0,\ \bar{P}t].$$

The material in the guide moves as a solid, so its velocity $v_L(t)$ is defined as $-dL/dt$ and corresponds to the (19) velocity multiplied by the area ratio (S_0/S_1). Then the length of the extruded part is

$$-L(t) = \int_0^t v_L(t)\, dt = -\bar{P}\int_0^t \left[1 - (1 - \rho_0(\bar{P}t))\exp\left(-\frac{t}{t_*}\right)\right]^{-1} dt. \tag{22}$$

From (19) and (22) we get that the length and velocity of the extruded part are dependent only on $\bar{P}$, which governs the hole resistance, and on the characteristic consilidation time t_*, which simplifies the inverse determination of the viscosity μ_1 or the parameters k, n, S_1/S_0, governing the hole resistance from measurements of L(t) or $v_L(t)$.

Sometimes, t_* has a negligible effect, e.g., if the extrusion time t_e is sufficiently large by comparison with $t_*(t_e \gg t_*)$, when (20) gives $v_L = -\bar{P}$. The speed V from (21) also tends to $-\bar{P}$. In essence, $t_e \gg t_*$ defines the condition for quasistationary extrusion, in

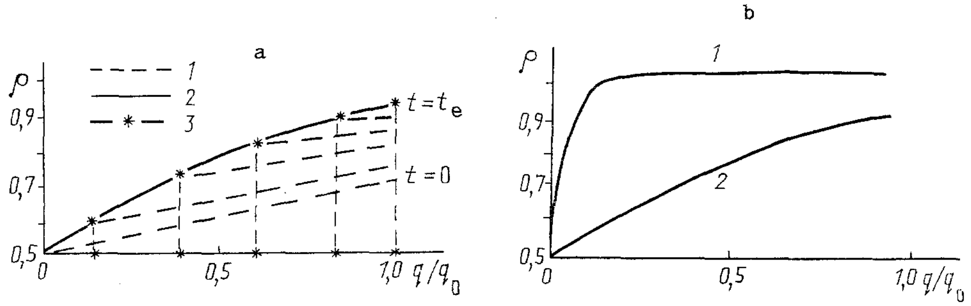

Fig. 2. Distribution of ρ in q as a function of: a) time (1 chamber, 2 rod, 3 boundary position); b) $\overline{P}$ (1, $\overline{P}$ = 0.025; 2, 0.25).

which the density does not vary in time ($\partial\rho/\partial t = 0$), and the mass flow rate is the same in all cross sections: ρV = const. For quasistationary conditions with low porosity, (22) gives $L(t) = \overline{P}t$, which simplifies determining the parameters appearing in the hole resistance.

Result Analysis. We assume that the density as a function of the mass coordinate at the start is linear:

$$\rho_0(q) = \rho_0 + (\rho_m - \rho_0)\, q/q_0. \tag{23}$$

If the actual distribution differs from (23), the latter can be considered as a linear interpolation for it. In Cartesian coordinates, (23) is $\rho_0(z) = \rho_0 \exp\left(\frac{\rho_m - \rho_0}{q_0} z\right)$.

Figure 2a shows $\rho(q, t)$ in the chamber (dashed lines) and in the guide (solid line) for various times with the following parameters: $P = 10^8$ Pa, $\rho_0 = 0.5$, $\rho_m = 0.7$, $S_0/S_1 = 36$, $q_0 = 4 \cdot 10^{-2}$ m. The points * on the q axis and on the $\rho(q)$ curves correspond to the positions of the time-varying boundary (mass coordinate $q = \overline{P}t$) separating the chamber and the guide.

One usually tends to meet the following two conditions during extrusion: 1) there is only a low density gradient along the extruded part, and 2) the density of much of the extruded rod is close to one, i.e., the specimen is substantially compacted.

Then Fig. 2b shows a good density distribution (curve 1) and a poor one (curve 2). The density variation for curve 1 occurs in a small part of the extruded rod (the mass of that part is less than 10% of the total), while the density throughout the rest is almost identical at 0.99 of the density for the incompressible matrix. For curve 2, the density is uniformly distributed along the length and the limiting density is only 0.86 of the density of the incompressible matrix. The initial density variation in these two cases is the same at 0.2, and the cases differ only in $\overline{P}$, which for the first curve is 0.025 and for the second is 0.25.

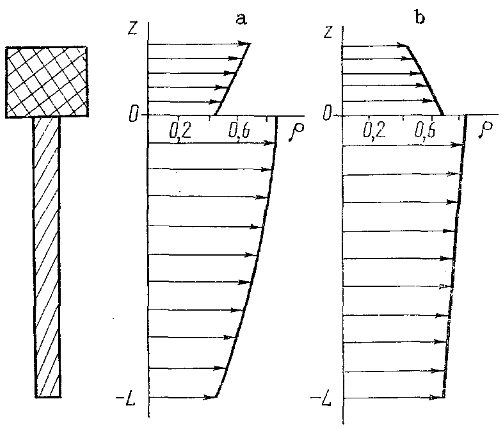

Fig. 3. Effects of initial density variation on the density distribution in the rod: a) $\Delta\rho = 0.2$, $\rho_0 = 0.5$, $\rho_m = 0.7$; b) $\Delta\rho = 0.2$, $\rho_0 = 0.7$, $\rho_m = 0.5$.

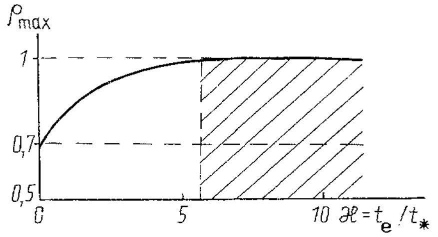

Fig. 4. Limiting density ρ_{max} as a function of κ.

It has been found [11, 12] that initial density variation has little effect on the final consolidation. This is due to density self-equalization. The density differences have a different effect on the extrusion. For a given difference in the initial distribution $\Delta\rho = |\rho_m - \rho_0| = 0.2$ but differing positions for the material (straight and reversed), one gets differing density distributions in the extruded material (Fig. 3a and b, correspondingly). One can say that for a given density difference, it is more favorable to have the bottom denser than the top (Fig. 3b). The reverse position leads to a substantial increase in the initial density difference (by a factor of two), while the density difference in the rod is reduced with the tablet reversed.

The basic dimensionless parameters governing the consolidation and extrusion are $\kappa = t_e/t_*$, which characterizes the ratio of the extrusion time t_e and consolidation time t_*, and $\bar{P}$, which characterizes the slot resistance ($\bar{P} = kP^nS_1/S_0\rho_1$). Figure 4 shows the limiting density ρ_{max} as a function of κ. It is monotone: for low $\kappa(\kappa \leq 1)$, the specimen is extruded without consolidation, and $\rho_{max} < 0.86$ for $\kappa = 5$, while the relative limiting density is close to one, and with $\kappa > 5$, there is merely an increase in the extrusion time without effect on the limiting density.

NOTATION

t, time; r and z, transverse and longitudinal coordinates; r_0 and H_0 tablet radius and height; r_1, hole radius; v, flow velocity; p_1 and μ_1, density and viscosity of incompressible matrix; ρ, relative density of material; μ and ξ, shear and bulk viscosities of material; σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, radial, tangential, and axial stresses; S_0 and S_1, cross-sectional area of chamber and guide; $\bar{P} = S_1kP^n/S_0\rho_1$, flow speed in hole; P, force on plunger; $q_0 = \int_0^{H(t)} \rho(z, t)dz + \bar{P}t$, relative tablet mass; L(t) and H(t) time dependence of rod length and tablet height; ρ_0 and ρ_m, densities of material in hole and at piston correspondingly at the initial instant; κ, dimensionless parameter, ratio of the characteristic consolidation and extrusion times; and t_e and t_c, characteristic extrusion and consolidation times.

LITERATURE CITED

1. Z. Tadmor and C. G. Gogos, Principles of Polymer Processing, Wiley, New York (1979).
2. G. A. Smirnov-Alyaev, Plastic-Strain Resistance in Materials [in Russian], Leningrad (1978).
3. L. M. Kachanov, The Principles of Plasticity Theory [in Russian], Moscow (1969).
4. A. V. Dubovik, "Heating in a non-Newtonian liquid on extrusion through a die," Inzh.-fiz. Zh., 39, No. 4, 710-715 (1980).
5. A. V. Dubovik and V. K. Bobolev, "Flow of a layer of non-Newtonian liquid on impact," Inzh.-fiz. Zh., 27, No. 2, 317-325 (1974).
6. G. Kh. Gessinger, Heat-Resisting Alloy Powder Metallurgy [in Russian], Chelyabinsk (1988).
7. A. G. Merzhanov, Self-Propagating High-Temperature Synthesis: 20 Years of Research and Discovery [in Russian], Preprint ISMAN SSSR, Chernogolovka (1989).
8. A. M. Stolin, Heat and Mass Transfer in Chemically Reacting Systems [in Russian], Minsk (1988), pp. 12-20.
9. V. V. Skorokhod, Rheological Principles of Sintering Theory [in Russian], Kiev (1972).
10. M. S. Koval'chenko, The Theoretical Principles of Hot Pressure Processing for Porous Materials [in Russian], Kiev (1980).
11. L. M. Buchatskii, A. M. Stolin, and S. I. Khudyaev, Poroshk. Metal., No. 9, 37-42 (1986).
12. A. M. Stolin, N. N. Zhilyaeva, and B. M. Khusid, Inzh.-fiz. Zh., 59, No. 2, 248-254 (1990).

13. L. M. Buchatskii and A. M. Stolin, Rheology and Polymer Processing Optimization [in Russian], Vol. 2, Izhevsk (1989), p. 107.
14. L. S. Stel'makh, N. N. Zhilyaeva, and A. M. Stolin, Inzh.-fiz. Zh., 61, No. 1, 33-40 (1991).

EQUIVALENCE OF FORMULATIONS OF PROBLEMS WHEN MODELING FLOWS OF RHEOLOGICALLY COMPLEX MEDIA IN SCREW-SHAPED CHANNELS

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UDC 536.24

The equivalence of two formulations of problems concerning the flow of a Newtonian liquid in a screw-shaped channel of an extruder – direct and inverse (rotation of jacket) – is analyzed.

The problem of the motion of a liquid in the screw-shaped channel of an extrusion machine is traditionally formulated as an inverse problem. In this formulation the screw is stationary and the casing rotates, and the problem is ultimately reduced to flow in a rectangular channel whose upper wall moves at an angle with respect to the longitudinal axis [1-3].

A different, direct formulation of the problem is also possible [4]. In this formulation the screw rotates and the casing is stationary. The solution of the problem in this case is obtained with the help of spiral coordinates introduced in a different manner. An example of such a formulation is given in [5].

Since the problem of accurate calculation of extruders (on which, by the way, there are many papers and monographs) is important, it is useful to study the relation between the two approaches to modeling.

When analyzing the direct formulation it should first be noted that in both [4] and [5] nonorthogonal spiral coordinate systems are introduced. In [6] it is proved that the velocity vector is self-similar relative to the third (spiral) coordinate. We first show that it is impossible to introduce orthogonal coordinates in which the spiral displacement is transformed into a translation of the coordinate.

Let $S_\alpha: R^3 \to R^3$ be a spiral displacement by an angle α (if the axis of the screw is taken as the Oz axis and the Ox and Oy axes are chosen to be orthogonal to the Oz axis, then this transformation has the form: $x \to x\cos\alpha - y\sin\alpha$, $y \to x\sin\alpha + y\cos\alpha$, $z \to z + \gamma\alpha$). The trajectory of a point M is the curve $\{S_\alpha M\}_{-\infty<\alpha<\infty}$ – the spiral line. We shall show that in a neighborhood of the point M it is possible to introduce an orthogoanl coordinate system so that the trajectories of the points in a neighborhood of M would be coordinate lines.

For this we show that there does not exist a surface orthogonal to the spiral lines in a neighborhood of the point M. This is an obvious consequence of Frobenius's theorem [7]. Here we shall give a direct proof.

We write the parametric equations of the spiral line passing through the point (x^0, y^0, z^0) as follows:

$$\begin{aligned} x &= x^0\cos\alpha - y^0\sin\alpha, \\ y &= x^0\sin\alpha + y^0\cos\alpha, \quad -\infty < \alpha < \infty, \\ z &= z^0 + \gamma\alpha, \end{aligned} \qquad \begin{cases} x(0) = x^0, \\ y(0) = y^0, \\ z(0) = z^0. \end{cases}$$

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